Multi-secretary problem with many types

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### Multi-secretary Problem

Given a hiring budget $B$ and horizon $T$, choose the top $B$ secretaries based on their realized abilities.

**Offline Problem:** Can see the entire future.

**Online Problem:** Non-anticipating.

### Common Heuristic

**OPT:**

\[
\max_{x_1, \ldots, x_T} \sum_{t=1}^T \theta_t x_t \quad \text{s.t.} \quad \sum_{t=1}^T x_t \leq B, x_t \in \{0, 1\}
\]

**Difficulty:** Online algorithm does not know the future i.e does not know all the $\theta_t$ in advance.

### Certainty Equivalent Principle

Replace the stochastic quantities by their expectations and solve the optimisation problem and use the solution.

For uniform distribution, CE is a **threshold** policy.

### Conservatism wrt Gaps

**Conservatism Principle**

If the CE threshold is close to a gap, use the gap as a threshold.

### Failure of CE Policy For Many Types w/ Gaps i.e CE incurs large regret

For the CE policy, there exists a distribution $F$ such that $\text{Regret}(B, T; \text{CE}) = \Theta(\sqrt{T})$.

### Universal Lower Bound i.e the best any online policy can do

Consider any $\beta \in [0, \infty)$ and $\varepsilon_0 \leq 1/2$. Then there exists a distribution $F_{\beta, \varepsilon_0}$ and a budget $B$ such that $\text{Regret}(B, T; \pi) = \Omega(T^{2/4 - 1/2}1\{\beta > 0\} + \log T \cdot 1\{\beta = 0\})$

### CwG Policy is near-optimal

For any $\beta \in [0, \infty)$ and $\varepsilon_0 \in [0, 1]$, suppose the distribution $F$ with associated gaps is $(\beta, \varepsilon_0)$-clustered. Then for all $T \in \mathbb{N}$ and for all $B \in [T]$, the regret of our CwG policy scales as

\[
\text{Regret}(B, T; \text{CwG}) = O\left(T^{1/2 - 1/2}1\{\beta > 0\} + (\log T)^2 \cdot 1\{\beta = 0\}\right)
\]

**Corollary:** If the distribution has a (small) discrete support, $\text{Regret}(B, T; \text{CwG}) \leq C\sqrt{\log(1/\varepsilon_0)}/\varepsilon_0$

### Numerical Simulations

#### Contributions

- **Analytical:** We introduce the class of $(\beta, \varepsilon_0)$-clustered distributions which subsume previously considered distributions. Identify $\beta$ as a key driver of the regret scaling. $\beta$ also captures the hardness of the problem.
- **Algorithmic:** Devise a new algorithmic principle called Conservatism wrt Gaps to deal with distributions which have gaps and achieve near optimal performance.
- **Extensions:** Our results also extend to the setting with many small types which are relevant to other NRM problems like order fulfillment.

### References

