Is Selling Complete Information (Approximately) Optimal?
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Main Result (2 States)

Result 1: In 2 states + m actions, \( OPT = FRev = O(m) \). The ratio is tight up to a constant.

**Important corollary:** The optimal menu has cardinality \( |\Omega(m)| \).

- Useful Lemma: Given any menu \( M \) with cardinality \( k \), we can achieve revenue \( \geq \equiv \) - Rev(M) by selling complete information.

Challenges and Proof Idea

- Selling Information vs. Auctions:
  - Different types interpret the signal differently ⇒ Buyer's value function for any fixed experiment is not linear in the type, but piecewise-linear.
  - IC constraints are more demanding.
  - Standard IC: After deviating to another type, the buyer has to follow the seller's recommendation (wlog assume the signal set is the action set).
  - IC-IC: Deviation from both the true type and the seller's recommendation.
  - "Double Deviation"
  - IR constraints are more demanding.
  - Initially, buyer has positive utility before receiving any information (max(0, 1 - \( \theta \)) in the example).

- Main Challenge: When there are 2 actions, \( IC \) is implied by standard IC + IR. Not the case for \( m \geq 3 \) actions.

Proof sketch for upper bound:
1. Relax the problem by dropping the \( s-IC \) constraints.
2. Show that the optimal menu of the relaxed problem has cardinality \( O(m) \).

- The "relaxed" optimal is achieved by a special class ~ "semi-informative menu"

\[
\begin{array}{cc}
(a) \text{ Pattern 1: } n_s \text{ and } n_m \text{ are fixed} & (b) \text{ Pattern 2: } n_s \text{ and } n_m \text{ are fixed} \\
\hline
\text{Pattern 1: } n_s \text{ and } n_m \text{ are fixed} & \text{Pattern 2: } n_s \text{ and } n_m \text{ are fixed} \\
\end{array}
\]

- By the useful lemma (above), \( OPT_{FRev} = O(m) \).

For the relaxed problem:
- We show a characterization of the optimal menu for the relaxed problem.
- Corollary: A sufficient condition under which selling complete information is optimal.

Open Questions

- For binary states, can we show an upper bound (in \( s \) actions) on the cardinality of the optimal menu?
- Other simple mechanisms (eg, semi-informative menu) that obtain a better approximation?

[ArXiv version of the paper: https://arxiv.org/abs/2202.09013]

Thank you for your patience!